Malaysia International Mathematics Olympiad Competition 2017

26 November, 2017
9.30 am – 11.00 am
- Upper Primary Paper -

Jointly Organised by :

Persatuan Matematik Olimpiad Malaysia (PERMATO)
Sekolah Jenis Kebangsaan (Cina) Lai Meng Bukit Jalil, Kuala Lumpur
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E Mathematics Olympiad System

马来西亚数学奥林匹克学会
黎明华文小学
力行国民型华文学校
数学奥林匹克学研中心
联合主办

Instructions:

- **Do not turn to the first page until you are told to do so.**
- **Write down your name, your contestant number and your school's name on the answer sheet.**
- **Write down all answers on the answer sheet. Only Arabic NUMERICAL answers are needed.**
- **Answer all 25 problems. The total is 120 points. For problems involving more than one answer, full credit will be given only if ALL answers are correct, no partial credit will be given. There is no penalty for a wrong answer.**
- **Diagrams shown may not be drawn to scale.**
- **No calculator or calculating device is allowed.**
- **Answer the problems with pencil, blue or black ball pen.**
- **All papers shall be collected at the end of this test.**

Section A (4 marks each)
A 组 (每题 4 分)

1. What is the 2017th digit after the decimal point of $0.20\dot{1}\dot{7}+0.201\dot{7}$
   （ Example : $0.0\dot{2}=0.0222…,\ 0.\dot{1}\dot{2}=0.1212…,\ 0.31\dot{6}=0.31666…$ ）

   试求 $0.20\dot{1}\dot{7}+0.201\dot{7}$ 的小数点后第 2017 位数字是多少？
   （ 例如 : $0.0\dot{2}=0.0222…,\ 0.\dot{1}\dot{2}=0.1212…,\ 0.31\dot{6}=0.31666…$ ）

2. Andy, Bella and Carol has each taken 2 cards from the 7 cards shown below. The digit sum of cards taken by Andy, Bella and Carol are 15, 7 and 9 respectively. What is the number on the remaining card?

   Andy、Bella、Carol 各取走七张卡片中的两张，Andy 取走两张数的数码相加等于 15，Bella 取走两张数的数码相加等于 7，Carol 取走两张数的数码相加等于 9，那么，剩的一张是什么号码？
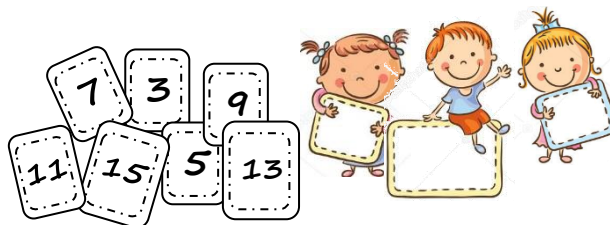
3. Given that： $8 \heartsuit 2 = 8 + 88 = 96$, $6 \heartsuit 4 = 6 + 66 + 666 + 6666 = 7404$, $9 \heartsuit$ $= 111105$,
    $= ?$

   已知： $8 \heartsuit 2 = 8 + 88 = 96$, $6 \heartsuit 4 = 6 + 66 + 666 + 6666 = 7404$, $9 \heartsuit$ $= 111105$, $= ?$

4. Divide a two-digit number by the sum of its digits, what's the maximum remainder?
   将一个二位数除以它的各位数字和，则得到的余数的最大值是多少？

5. A and B each has some money, money with A is 4 times of money with B. If A uses one third of his money, one third of the remaining and then being given \$7.00, money left eventually will be the same as money with B. How much money does A have ?

   A、B 两人各有钱若干，已知 A 的钱数是 B 的 4 倍，当 A 用去 $\frac{1}{3}$ 后，又用去余下的 $\frac{1}{3}$，如果这时 A 给 B \$7.00，A 、B 两人的钱数正好相等。A 原来有多少钱？

6. I have a deck of cards from which some are missing. If I deal them equally between nine people, I have two cards to spare. If I deal them equally between four people, I have three cards to spare. If I deal them between seven people, I have five cards to spare. There are normally 52 cards in the deck. How many are missing?

我有一副不完整的扑克牌。如果平均分给 9 个人，还剩两张；如果分给 4 个人，还剩三张；如果分给 7 个人，则剩五张。一副扑克牌通常有 52 张。请问这副不完整的扑克牌不见了多少张牌？

7. ABCD is a trapezium with AB parallel to DC, AB = 8 cm, DC=22 cm and the area of triangle ABP is 24 cm². What is the area of the trapezium ABCD in square centimetres?

   梯形 ABCD，AB 平行 DC，AB 长 8cm，DC 长 22cm，三角形 ABP 的面积是 24cm² ，求此梯形的面积。
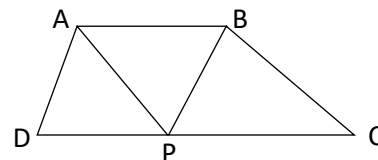
8. Given that A is a positive integer， both A and $A^3 + 5$ are prime numbers. Find the value of $A^5 + 2017 = $?

   已知 A 是正整数，且 A 与 $A^3 + 5$ 都是质数，那么，$A^5 + 2017 = $?

9. Let a, b, c, d, e be consecutive positive integers such that $b + c + d$ is a perfect square and $a + b + c + d + e$ is a perfect cube. Find the smallest possible value of c.

   设 a，b，c，d，e 是连续的正整数，使得 $b + c + d$ 是一个完全平方数, $a + b + c + d + e$ 是一个完全立方数。 求 c 的最小可能值。

10. Five persons, A, B, C, D and E, raised the same amount of money to buy the same gifts together. Four of them A, B, C and D bought 8 gifts, 6 gifts, 12 gifts and 9 gifts more than E respectively. When they settled the bill, A paid $7 to B. How much did C and D pay to E respectively?

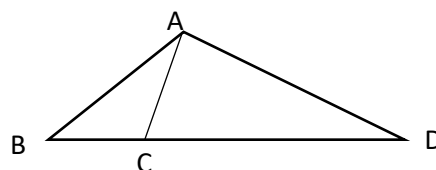   A、B、C、D、E 五人凑同样多的钱一同去礼品店买同一种礼物，而且 A、B、C、D 分别比 E 多买了 8、6、12、9 件礼物，最后大家结算时，A 付$7.00 给 B，那么 C、D 应分别付给 E 多少钱？

Section B (5 marks each)
B 组 (每题 5 分)

11. In the diagram below, given that AB = CD， ∠BAC = 30°， ∠ABC = 40°. What is ∠ADC?

    如图，ΔABD 中，已知 AB = CD, ∠BAC = 30°, ∠ABC = 40°, 求 ∠ADC.

12. The weather is getting colder, the grass on the field is decreasing at a constant rate every day. Mr McDonald told that the grass on the field could feed 20 goats for 5 days or 16 goats for 6 days. If Mr McDonald has 11 goats, the grass could last for how many days?

    天气越来越冷，农场的牧草每天以均匀的速度减少了。麦当劳叔叔计算牧场上的草可供 20 头羊吃 5 天，或者可供 16 头羊吃 6 天。如果麦当劳叔叔现有 11 头羊，可以吃几天？

13. The school has 4 co-curricular activities for the kids to choose, these are Olympiad Mathematics, Science, Chinese and English. The kids could at most choose 2 activities or they may opt not to attend any of the activities. What is the minimum number of primary 5 kids if there are only 2 kids having the same combinations of co-curricular activities?
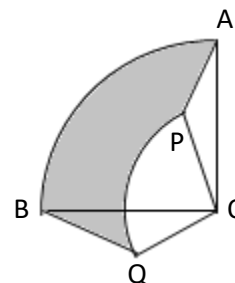
    学校有奥数、科学、华文和英文四种课外活动供选择，五年级学生每位最多可以参加两项，也可以不参加，若要保证有两个人参加的活动情形完全相同，至少要有多少位五年级的学生？

14. In the diagram below, the radius of quadrant OPQ is 7 cm and the radius of quadrant OAB is 14 cm. Given that ∠AOP = 30°, Find the area of the shaded region ABQP.
    ($\pi = \frac{22}{7}$)
    下图中，OPQ 是半径为 7 cm 的四分之一圆，OAB 是半径为 14 cm 的四分之一圆.
   已知 ∠AOP = 30°，求阴影部分 ABQP 的面积。($\pi = \frac{22}{7}$)

15. The siblings are counting money in their saving. The elder brother has $18 more than $\frac{9}{7}$ of the elder sister's; while the younger brother has $108 more than $\frac{2}{3}$ of the elder brother's; the younger sister's saving is $20 more than $\frac{9}{8}$ of the younger brother's. Four of them have a total saving of $2018. How much money does the elder sister has in her saving?

    每个人都把存钱筒里的钱倒出来，数一数，哥哥存的钱是姐姐的 $\frac{9}{7}$ 倍多$18，弟弟的钱是哥哥的 $\frac{2}{3}$ 倍多$108，妹妹的钱是弟弟的 $\frac{9}{8}$ 倍多$20，已知四个人的钱共有$2018，则姐姐有多少钱？

16. In the figure below, how many squares in any sizes are there, in which the number of black square units is the same as the number of white square units?

    下图中有多少个正方形，可以是任意边长但是正方形内的黑色格子数量要等于白色格子？
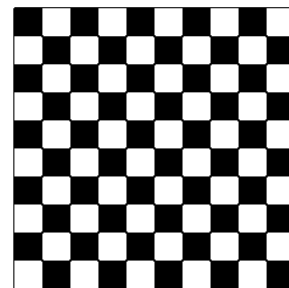
17. 2017 kg apples were purchased at the price of $2.50 per kg in a fruit shop. The apples are sold at a profit of 40%. When 92% of the apples were sold, the rest of the apples were sold at a discounted price. As a result, the profit is 98% of the estimated value. What is the ratio of the discounted selling price of the apples to the cost price of the apples?

    水果店购进单价为$2.50 的苹果 2017 千克，按 40%的利润定价出售，当售出 92%的苹果后，剩下的苹果打折出售，结果获利是预定利润的 98%，那么剩下部分的苹果出售价是购进价的几分之几？

18. The cubic of any positive integer n that is bigger than 1 may be expressed as the sum of n contiguous odd numbers. For example, $2^3 = 3+5$, $3^3 = 7+9+11$, $4^3 =13+15+17+19$, …. As such, if n^3 is expressed as the sum of n contiguous odd numbers, and one of them is 2017, what is the value of n?

    任意大于 1 的正整数 n 的三次方均可表示成 n 个连续奇数的和，如：$2^3 = 3+5$，$3^3 = 7+9+11$，$4^3 =13+15+17+19$，…，按这个规律，若 n^3 表示成 n 个连续奇数的和时，其中有一个奇数是 2017，那么 n 的值是多少？

19. a, b and c are different natural numbers between 0 and 9. a, b and c can form 6 different 3-digit numbers. If the sum of the five 3-digit numbers is 2017, what is the rest 3-digit number?

    a、b、c 分别是自然数 0 至 9 中不同的数字，用 a、b、c 一共可以组成 6 个不同数字的三位数，如果其中五个三位数的和是 2017，那么另一个三位数是多少？

20. Evaluate $2018 \times (\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + …… + \frac{1}{3^{2017}} + \frac{1}{2 \times 3^{2017}})$.

    计算：$2018 \times (\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + …… + \frac{1}{3^{2017}} + \frac{1}{2 \times 3^{2017}})$

Section C (6 marks each)
C 组 (每题 6 分)

21. We need to put 2017 Ping-pong balls into 6 boxes which are labelled a_1, a_2, a_3, a_4, a_5 and a_6, being arranged from the smallest to the biggest boxes. It is known that a bigger box could hold twice as many balls as a smaller one, that is $2a_1 = a_2$. We try to put as many balls as possible into the 6 boxes, how many balls are there in the a_6 box?

要把 2017 个乒乓球放入编号为 a_1, a_2, a_3, a_4, a_5 和 a_6 的六个从小排到大的箱子里，已知编号大 1 的箱子能装乒乓球的数量是编号小 1 的箱子的 2 倍，即 $2a_1 = a_2$，若要尽可能多的把乒乓球放入 6 个箱子里，那么编号为 a_6 的箱子里放了多少个乒乓球？

22. As shown in the diagram below, ΔACD, is isosceles, with $AD = CD$, and $\angle ADC = 110°$. Given also that PT is parallel to AC, the points Q, R and S are on PT and B on AC are such that $AB = PQ = QR = RS = ST$. If $\angle DAT = 67°$ and PA is perpendicular to AC, find $\angle AQB + \angle ARB + \angle ASB + \angle ATB$ in degrees.

如图所示，ΔACD是一等腰三角形，$AD = CD$, 且 $\angle ADC = 110°$. 已知 PT 平行 AC。 Q, R 和 S 为 PT 线上的点， 点 B 在 AC 上，$AB = PQ = QR = RS = ST$。若 $\angle DAT = 67°$ 且 PA 垂直 AC, 求 $\angle AQB + \angle ARB + \angle ASB + \angle ATB$ 。
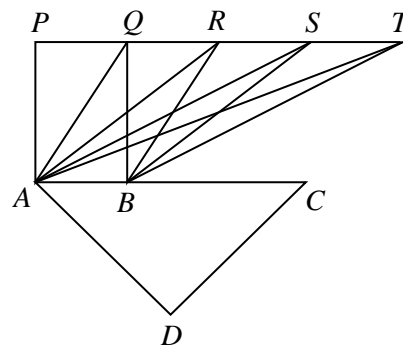
23. Solve the equation $\dfrac{x-3}{2017} + \dfrac{x-5}{2015} + \dfrac{x-7}{2013} = \dfrac{x-2017}{3} + \dfrac{x-2015}{5} + \dfrac{x-2013}{7}$. $x = ?$

解方程式 $\dfrac{x-3}{2017} + \dfrac{x-5}{2015} + \dfrac{x-7}{2013} = \dfrac{x-2017}{3} + \dfrac{x-2015}{5} + \dfrac{x-2013}{7}$ ， $x = ?$
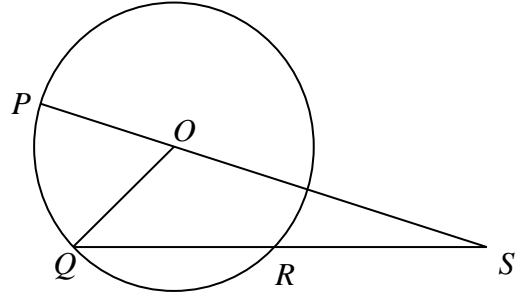
24. Mr. and Mrs. Lim are going for a picnic with their baby. They will need 3 jars of baby food for the picnic. Their pantry is stocked with 2 jars of different baby fruits, 4 jars of different baby vegetables, and 3 jars of different baby meat. If they select 3 jars at random, what is the probability that one type of the baby food is not being selected (i.e. there is either no fruit, no vegetable, or no meat)?

林先生和林太太带着婴儿去野餐， 他们要在 2 罐不同的婴儿果汁，4 罐不同的婴儿蔬菜和 3 罐不同的婴儿肉品中挑选 3 罐。如果在随机选择下，漏选其中一项食品的概率是多少（例如：没有选到水果的，

6

或没有蔬菜的，或没有肉类的）？

25. In the diagram, P, Q and R are 3 points on the circle with O as its centre. The lines PO and QR are extended to meet at S. Suppose that $RS = OP$, $\angle PSQ = 14°$ and $\angle POQ = x°$. Find the value of x in degrees.

如图所示，P, Q 和 R 是圆 O 上的 3 个点，O 为圆心。PO 与 QR 的延长线相交于 S。若 $RS = OP$，且 $\angle PSQ = 14°$，$\angle POQ = x°$. 求 x .